Rice's Theorem. *Let $\mathcal{L}$ be any set of languages that satisfies the following conditions:*
- *There is a Turing machine Y such that $\text{Accept}(Y) \in \mathcal{L}$.*
- *There is a Turing machine N such that $\text{Accept}(N) \notin \mathcal{L}$.*

The language $\text{AcceptIn}(\mathcal{L}) := \big\{ \langle M \rangle \ \big| \ \text{Accept}(M) \in \mathcal{L} \big\}$ is undecidable.

Prove that the following languages are undecidable *using Rice's Theorem*:

1. $\text{AcceptRegular} := \big\{ \langle M \rangle \ \big| \ \text{Accept}(M) \text{ is regular} \big\}$

2. $\text{AcceptIllini} := \big\{ \langle M \rangle \ \big| \ M \text{ accepts the string } \texttt{ILLINI} \big\}$

3. $\text{AcceptPalindrome} := \big\{ \langle M \rangle \ \big| \ M \text{ accepts at least one palindrome} \big\}$

4. $\text{AcceptThree} := \big\{ \langle M \rangle \ \big| \ M \text{ accepts exactly three strings} \big\}$

5. $\text{AcceptUndecidable} := \big\{ \langle M \rangle \ \big| \ \text{Accept}(M) \text{ is undecidable} \big\}$

To think about later. Which of the following are undecidable? How would you prove that?

1. $\text{Accept}\{\{\varepsilon\}\} := \big\{ \langle M \rangle \ \big| \ M \text{ accepts only the string } \varepsilon; \text{ that is, } \text{Accept}(M) = \{\varepsilon\} \big\}$

2. $\text{Accept}\{\varnothing\} := \big\{ \langle M \rangle \ \big| \ M \text{ does not accept any strings; that is, } \text{Accept}(M) = \varnothing \big\}$

3. $\text{Accept}\varnothing := \big\{ \langle M \rangle \ \big| \ \text{Accept}(M) \text{ is not an acceptable language} \big\}$

4. $\text{Accept=Reject} := \big\{ \langle M \rangle \ \big| \ \text{Accept}(M) = \text{Reject}(M) \big\}$

5. $\text{Accept}\neq\text{Reject} := \big\{ \langle M \rangle \ \big| \ \text{Accept}(M) \neq \text{Reject}(M) \big\}$

6. $\text{Accept}\cup\text{Reject} := \big\{ \langle M \rangle \ \big| \ \text{Accept}(M) \cup \text{Reject}(M) = \Sigma^* \big\}$