CS 473: Undergraduate Algorithms, Spring 2010
Homework 8

Written solutions due Tuesday, April 20, 2010 in class.

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1. Suppose you have already computed a maximum (s,t)-flow f in a flow network G with integer capacities. Let k be an arbitrary positive integer, and let e be an arbitrary edge in G whose capacity is at least k.

   (a) Suppose we *increase* the capacity of e by k units. Describe and analyze an algorithm to update the maximum flow.

   (b) Now suppose we *decrease* the capacity of e by k units. Describe and analyze an algorithm to update the maximum flow.

   For full credit, both algorithms should run in $O(Ek)$ time. *[Hint: First consider the case $k = 1$.]*

2. Suppose we are given an array $A[1..m][1..n]$ of non-negative real numbers. We want to *round* A to an integer matrix, by replacing each entry x in A with either $\lfloor x \rfloor$ or $\lceil x \rceil$, without changing the sum of entries in any row or column of A. For example:

$$\begin{bmatrix} 1.2 & 3.4 & 2.4 \\ 3.9 & 4.0 & 2.1 \\ 7.9 & 1.6 & 0.5 \end{bmatrix} \longmapsto \begin{bmatrix} 1 & 4 & 2 \\ 4 & 4 & 2 \\ 8 & 1 & 1 \end{bmatrix}$$

   Describe an efficient algorithm that either rounds A in this fashion, or reports correctly that no such rounding is possible.

3. A *cycle cover* of a given directed graph $G = (V, E)$ is a set of vertex-disjoint cycles that cover all the vertices. Describe and analyze an efficient algorithm to find a cycle cover for a given graph, or correctly report that none exists. *[Hint: Use ipartite atching!]*