∽ New CS 473: Algorithms, Spring 2015 ∾
Homework 1

Due Tuesday, February 3, 2015 at 5pm

For this and all future homeworks, groups of up to three students can submit joint solutions. Please print (or typeset) the name and NetID of every group member on the first page of each submission.

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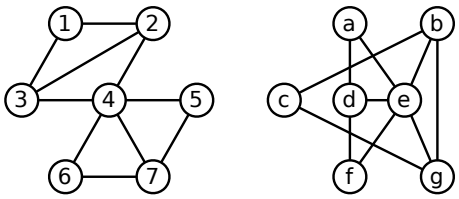
1. Two graphs are ***isomorphic*** if one can be transformed into the other just by relabeling the vertices. For example, the graphs shown below are isomorphic; the left graph can be transformed into the right graph by the relabeling $(1, 2, 3, 4, 5, 6, 7) \mapsto (c, g, b, e, a, f, d)$.



Two isomorphic graphs.

Consider the following related decision problems:

- GraphIsomorphism: Given two graphs G and H, determine whether G and H are isomorphic.
- EvenGraphIsomorphism: Given two graphs G and H, such that every vertex in G and every vertex in H has even degree, determine whether G and H are isomorphic.
- SubgraphIsomorphism: Given two graphs G and H, determine whether G is isomorphic to a subgraph of H.

(a) Describe a polynomial-time reduction from EvenGraphIsomorphism to GraphIsomorphism.

(b) Describe a polynomial-time reduction from GraphIsomorphism to EvenGraphIsomorphism.

(c) Describe a polynomial-time reduction from GraphIsomorphism to SubgraphIsomorphism.

(d) Prove that SubgraphIsomorphism is NP-complete.

(e) What can you conclude about the NP-hardness of GraphIsomorphism? Justify your answer.

[Hint: These are all easy!]

2. Prove that the following problems are NP-hard.

   (a) Given an undirected graph G, does G have a spanning tree with at most 473 leaves?

   (b) Given an undirected graph $G = (V, E)$, what is the size of the largest subset of vertices $S \subseteq V$ such that at most 2015 edges in E have both endpoints in S?

3. The Hamiltonian cycle problem has two closely related variants:

   - UNDIRECTEDHAMCYCLE: Given an *undirected* graph G, does G contain an *undirected* Hamiltonian cycle?
   - DIRECTEDHAMCYCLE: Given an *directed* graph G, does G contain a *directed* Hamiltonian cycle?

   This question asks you to prove that these two problems are essentially equivalent.

   (a) Describe a polynomial-time reduction from UNDIRECTEDHAMCYCLE to DIRECTEDHAMCYCLE.

   (b) Describe a polynomial-time reduction from DIRECTEDHAMCYCLE to UNDIRECTEDHAMCYCLE.

*4. ***[Extra Credit]*** Describe a direct polynomial-time reduction from 4COLOR to 3COLOR. (This is a lot harder than the opposite direction!)

New CS 473 Spring 2015 — Homework 1 Problem 1

Name:	NetID:
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(a) Describe a polynomial-time reduction from EvenGraphIsomorphism to GraphIsomorphism.

(b) Describe a polynomial-time reduction from GraphIsomorphism to EvenGraphIsomorphism.

(c) Describe a polynomial-time reduction from GraphIsomorphism to SubgraphIsomorphism.

(d) Prove that SubgraphIsomorphism is NP-complete.

(e) What can you conclude about the NP-hardness of GraphIsomorphism? Justify your answer.

New CS 473 Spring 2015 — Homework 1 Problem 2

Name:	NetID:
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(a) Given an undirected graph G, does G have a spanning tree with at most 473 leaves?

(b) Given an undirected graph $G = (V, E)$, what is the size of the largest subset of vertices $S \subseteq V$ such that at most 2015 edges in E have both endpoints in S?

New CS 473 Spring 2015 — Homework 1 Problem 3

Name:	NetID:
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Name:	NetID:

(a) Describe a polynomial-time reduction from UNDIRECTEDHAMCYCLE to DIRECTEDHAMCYCLE.

(b) Describe a polynomial-time reduction from DIRECTEDHAMCYCLE to UNDIRECTEDHAMCYCLE.

New CS 473 Spring 2015 — Homework 1 Problem 4

Name:	NetID:
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Submit in the same box as Problem 1 (but don't staple problems 1 and 4 together)

[Extra Credit] Describe a direct polynomial-time reduction from 4COLOR to 3COLOR.
