☙ New CS 473: Algorithms, Spring 2015 ❧
Homework 9

Due Tuesday, April 21, 2015 at 5pm

All homework must be submitted electronically via Moodle as separate PDF files, one for each numbered problem. Please see the course web site for more information.

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1. Suppose we are given an array $A[1..m][1..n]$ of real numbers. We want to *round* A to an integer array, by replacing each entry x in A with either $\lfloor x \rfloor$ or $\lceil x \rceil$, without changing the sum of entries in any row or column of A. For example:

$$\begin{bmatrix} 1.2 & 3.4 & 2.4 \\ 3.9 & 4.0 & 2.1 \\ 7.9 & 1.6 & 0.5 \end{bmatrix} \longmapsto \begin{bmatrix} 1 & 4 & 2 \\ 4 & 4 & 2 \\ 8 & 1 & 1 \end{bmatrix}$$

   Describe and analyze an efficient algorithm that either rounds A in this fashion, or reports correctly that no such rounding is possible.

2. Suppose we are given a set of boxes, each specified by their height, width, and depth in centimeters. All three side lengths of every box lie strictly between 10cm and 20cm. As you should expect, one box can be placed inside another if the smaller box can be rotated so that its height, width, and depth are respectively smaller than the height, width, and depth of the larger box. Boxes can be nested recursively. Call a box *visible* if it is not inside another box.

   (a) Describe and analyse an algorithm to find the largest subset of the given boxes that can be nested so that only one box is visible.

   (b) Describe and analyze an algorithm to nest *all* the given boxes so that the number of visible boxes is as small as possible. *[Hint: Do **not** use part (a).]*

3. Describe and analyze an algorithm for the following problem, first posed and solved by the German mathematician Carl Jacobi in the early 1800s.[1]

   > *Disponantur nn quantitates $h_k^{(i)}$ quaecunque in schema Quadrati, ita ut k habeantur n series horizontales et n series verticales, quarum quaeque est n terminorum. Ex illis quantitatibus eligantur n transversales, i.e. in seriebus horizontalibus simul atque verticalibus diversis positae, quod fieri potest $1.2 \ldots n$ modis; ex omnibus illis modis quaerendum est is, qui summam n numerorum electorum suppeditet maximam.*

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[1] Carl Gustav Jacob Jacobi. De investigando ordine systematis aequationum differentialum vulgarium cujuscunque. *J. Reine Angew. Math.* 64(4):297–320, 1865. Posthumously published by Carl Borchardt.

For the few students who are not fluent in mid-19th century academic Latin, here is a modern English translation of Jacobi's problem. Suppose we are given an $n \times n$ matrix M. Describe and analyze an algorithm that computes a permutation σ that maximizes the sum $\sum_i M_{i,\sigma(i)}$, or equivalently, permutes the columns of M so that the sum of the elements along the diagonal is as large as possible.

Please do not submit your solution in mid-19th century academic Latin.

New CS 473 Spring 2015 — Homework 9 Problem 1

Name:	NetID:
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Describe and analyze an algorithm to round a given array $A[1..m][1..n]$ of real numbers to an integer array, without changing the sum of entries in any row or column.

New CS 473 Spring 2015 — Homework 9 Problem 2

Name:	NetID:
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(a) Describe and analyse an algorithm to find the largest subset of the given boxes that can be nested so that only one box is visible.

(b) Describe and analyze an algorithm to nest *all* the given boxes so that the number of visible boxes is as small as possible. *[Hint: Do **not** use part (a).]*

New CS 473 Spring 2015 — Homework 9 Problem 3

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Disponantur nn quantitates $h_k^{(i)}$ *quaecunque in schema Quadrati, ita ut k habeantur n series horizontales et n series verticales, quarum quaeque est n terminorum. Ex illis quantitatibus eligantur n transversales, i.e. in seriebus horizontalibus simul atque verticalibus diversis positae, quod fieri potest* $1.2\ldots n$ *modis; ex omnibus illis modis quaerendum est is, qui summam n numerorum electorum suppeditet maximam.*
